Math 3B — Week 1

Integrals by Definition For each of the following, write the integral as a Riemann sum or determine what integral the Riemann sum is describing.

(a) $\displaystyle\int_0^\pi x^2 \tan(x)\, dx$

(b) $\displaystyle\int_1^2 x^2 + x^3\, dx$

(c) $\displaystyle\int_2^4 \left(x + x^2\right)^2\, dx$

(d) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^n \frac{2}{n}\left(3 - \left(5 + \frac{2i}{n}\right)^2\right)$

(e) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^n \frac{e^{i/n}}{n + i}$

(f) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^n -\frac{5\sin\left(-\frac{5i}{n}\right)}{n}$

Integrals as Areas Evaluate each of the following integrals geometrically.

(a) $\displaystyle\int_0^1 x\,dx$

(b) $\displaystyle\int_0^1 1\,dx$

(c) $\displaystyle\int_0^1 -1\,dx$

(d) $\displaystyle\int_1^0 1\,dx$

(e) $\displaystyle\int_1^0 -1\,dx$

(f) $\displaystyle\int_0^{2\pi} \sin(x)\,dx$